

單元 計算機的應用(三角函數)

討論一 $\sin \theta$ 、 $\cos \theta$ 、 $\tan \theta$

(1)用計算機求： $\sin 40^\circ$ ， $\cos 40^\circ$ 的值。

(2)度數與弧度的互換，求：① $45^\circ = ?$ ② $18^\circ = ?$ ③ $\frac{\pi}{3} = ?$ ④ $4 = ?$

(3)用計算機驗證：

$$\textcircled{1} \sin 870^\circ = \sin 30^\circ$$

$$\textcircled{2} \sin(-1215^\circ) = -\sin 45^\circ$$

$$\begin{aligned} -\sin(-1215^\circ) &= -\sin(3 \times 360^\circ + 135^\circ) = -\sin 135^\circ = \\ &= -\sin(180^\circ - 45^\circ) = -\sin 45^\circ \end{aligned}$$

(4)求下列三角函數的近似值(求到小數第二位)：

$$\textcircled{1} \sin \frac{\pi}{6} \times \cos \frac{\pi}{3} = ? \quad \textcircled{2} \frac{\sin 2 + \cos 3}{\tan 5} = ?$$

討論二 反三角函數的求法

(1)求下列各式的值：

$$\textcircled{1} \sin \theta = \frac{1}{2}, \theta = ? \quad \textcircled{2} \tan \phi = 1, \phi = ? \quad \textcircled{3} \cos x = \frac{3}{5}, x = ?$$

(2)求下列各式的值：

$$\textcircled{1} \sin^{-1} 1 = ? \quad \textcircled{2} \tan^{-1}(-1) = ? \quad \textcircled{3} \cos^{-1} \frac{3}{2} = ? \quad \textcircled{4} \cos^{-1} \left(-\frac{1}{3}\right) = ?$$

討論三 三角函數的應用

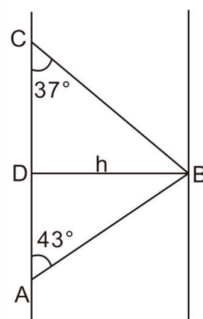
(1)右圖中 $\angle CAB = 43^\circ$, $\angle BCA = 37^\circ$, $\overline{AC} = 12 \text{ m}$, 求河寬 $\overline{BD} = ?$

$$\frac{h}{\overline{AD}} = \tan 43^\circ, \overline{AD} = \frac{h}{\tan 43^\circ}$$

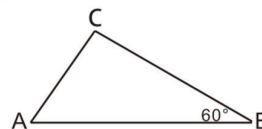
$$\frac{h}{\overline{CD}} = \tan 37^\circ, \overline{CD} = \frac{h}{\tan 37^\circ}$$

$$\overline{AD} + \overline{CD} = \frac{h}{\tan 43^\circ} + \frac{h}{\tan 37^\circ} = 12$$

$$h = \frac{12 \tan 43^\circ \tan 37^\circ}{\tan 43^\circ + \tan 37^\circ} =$$



- (2) 比薩斜塔高 184.5 呎，某人欲知其傾斜程度，在離塔 123 呎處測得塔頂的仰角 60° ，如圖求 $\angle CAB = ?$ (以六十進位表示)



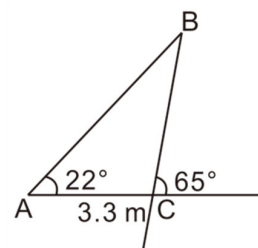
$$\frac{\sin C}{123} = \frac{\sin B}{184.5} \quad \sin C = \frac{123 \sin 60^\circ}{184.5} = \frac{123 \times \frac{\sqrt{3}}{2}}{184.5} = 0.5774$$

反查 $\angle C =$

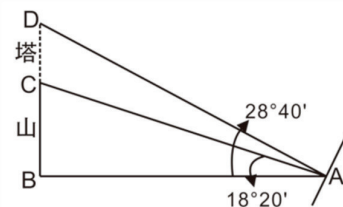
$$\angle A = 180^\circ - (\angle B + \angle C) = 180^\circ - (60^\circ + 35^\circ 16') =$$

- (3) 如圖，求 $\overline{AB} = ?$ (求到小數第三位)

$$\angle B = 65^\circ - 22^\circ = 43^\circ \quad \frac{3.3}{\sin 43^\circ} = \frac{\overline{AB}}{\sin 115^\circ}$$



- (4) 一塔高 125 m，立在河邊的山岩上，從塔頂俯視俯為 $28^\circ 40'$ ，而從塔底俯看對岸同點，俯角為 $18^\circ 20'$ ，試求山岩及河寬？



$$\frac{x}{\overline{AB}} = \tan 18^\circ 20' \quad \frac{x + 125}{\overline{AB}} = \tan 28^\circ 40'$$

$$\frac{x}{x + 125} = \frac{\tan 18^\circ 20'}{\tan 28^\circ 40'}$$

$$\tan 18^\circ 20' x + 125 + \tan 18^\circ 20' = x \tan 28^\circ 40'$$

$$\text{山高} = x = \frac{125 + \tan 18^\circ 20'}{\tan 28^\circ 40' - \tan 18^\circ 20'} \quad \frac{\text{山高}}{\overline{AB}} = \tan 18^\circ 20'$$

$$\overline{AB} = \frac{\text{山高}}{\tan 18^\circ 20'} =$$

討論四 直角坐標和極坐標的轉換

(1)將下列極坐標換成直角坐標(x, y),

① $(2, \frac{\pi}{2})$

$$\begin{aligned}x &= r \cos \theta \\y &= r \sin \theta\end{aligned}$$

② $(6, 60^\circ)$

$$\begin{aligned}x &= r \cos \theta \\y &= r \sin \theta\end{aligned}$$

(2)將右列直角坐標換成極坐標。 ① $(1, \sqrt{3})$ ② $(6, 6)$

(3) $y = \sqrt{3} \sin 30^\circ + \cos 30^\circ = 2 \sin \theta$, 求 $\theta = ?$

$$\begin{aligned}y &= 2 \left(\frac{\sqrt{3}}{2} \sin 30^\circ + \frac{1}{2} \cos 30^\circ \right) \\&= 2(\cos 30^\circ \times \sin 30^\circ + \sin 30^\circ \cos 30^\circ) \\&= 2 \sin 60^\circ\end{aligned}$$

討論五 直線的交角

(1) $L_1 : 2x - y + 30 = 0$ 與 $L_2 : x + y + 1 = 0$ 之所夾的銳角為 θ ，
求 $\cos 2\theta = ?$

$$\tan \theta_1 = m_1 = -2 / -1 = 2$$

$$\tan \theta_2 = m_2 = -1/1 = -1$$

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \frac{2 + 1}{1 - 2} = \pm 3 (\text{取正})$$

(2) $L_1 : 2x + 2y + 7 = 0$ 與 $L_2 : \sqrt{3}x - y + 6 = 0$ 之所夾鈍角為 θ ，
求 $\cos 2\theta = ?$

$$\tan \theta_1 = -1, \tan \theta_2 = \sqrt{3}, \tan \theta = \pm \frac{\sqrt{3} + 1}{1 - \sqrt{3}} (\text{取負})$$